

# Quantum mechanics, Furry's hypothesis and a measure of decoherence in the $K^0\bar{K}^0$ system

R.A. Bertlmann, W. Grimus and B.C. Hiesmayr  
*Institut für Theoretische Physik, Universität Wien*  
*Boltzmanngasse 5, A-1090 Vienna, Austria*

## Abstract

We consider strangeness correlations of the EPR type in  $K^0\bar{K}^0$  pairs created in a  $J^{PC} = 1^{--}$  state as a function of time under the hypothesis that spontaneous decoherence takes place. We parameterize the degree of decoherence by a factor  $(1 - \zeta)$  which multiplies the quantum-mechanical interference terms occurring in the amplitudes for like and unlike strangeness events and discuss the dependence of this procedure on the basis chosen in the  $K^0\bar{K}^0$  space to which the interference terms correspond. Consequently, all statements about the “decoherence parameter”  $\zeta$  inferred from experimental data are basis-dependent as well. We illustrate this point by estimating the value of  $\zeta$  for the two bases  $\{K_L, K_S\}$  and  $\{K^0, \bar{K}^0\}$  with the help of recent data of the CPLEAR experiment.

## I. INTRODUCTION

Since the first formulation of the EPR paradox in 1935 [1] tests of quantum mechanics (QM) against local realistic (hidden variable) theories are of great interest. According to QM, if a pair of particles is created by any kind of interaction in an entangled state, the two particle wave function retains its non-separable character even if the particles are space-like separated. The famous inequality of J.S. Bell [2] made it possible to discriminate quantitatively between the predictions of QM and of local realistic theories. Numerous experiments have been performed since, in particular, using entangled photon states, which all confirmed QM (see, e.g., Refs. [3,4]). On the particle physics side, entangled  $K^0\bar{K}^0$  and  $B^0\bar{B}^0$  states are suitable objects for the study of EPR-like correlations [5–8]. The presence of the QM interference term can also be deduced from existing data on  $B^0\bar{B}^0$  systems produced in the decay of  $\Upsilon(4S)$  as was demonstrated in Refs. [9–11]. Recently, an important experiment of the CPLEAR Collaboration using strangeness correlations of  $K^0\bar{K}^0$  pairs created by annihilation of  $p\bar{p}$  pairs at rest showed impressively the non-separability of the QM  $K^0\bar{K}^0$  wave function in this situation [12]. This experiment excluded a spontaneous wave function factorization immediately after the  $K^0\bar{K}^0$  creation with a CL of more than 99.99%.

In this paper we want to draw the attention to the fact that the notion of spontaneous factorization depends on the basis chosen in the two systems which, after the creation of the particle pairs, are non-interacting. The reason for this is that a basis has to be fixed with respect to which the process of spontaneous factorization takes place [13] (see also Refs. [10,11]). In our case of interest the two non-interacting systems are given by the two neutral kaons moving into opposite directions in the rest frame of the  $K^0\bar{K}^0$  source. In this paper we discuss spontaneous factorization and decoherence of the  $K^0\bar{K}^0$  wave function created by a  $J^{PC} = 1^{--}$  state as a function of the basis chosen to describe the non-interacting neutral kaons on each side of the source. We modify the quantum-mechanical interference term of the entangled 2-kaon state by multiplying it with  $(1 - \zeta)$ . Thus we change QM expressions in such a way that the decoherence parameter  $\zeta$  [14] parameterizes the deviation from QM (corresponding to  $\zeta = 0$ ) and gives a measure how far the total system is away from total decoherence or spontaneous factorization ( $\zeta = 1$ ). As an observable relevant in this discussion we use the time-dependent asymmetry  $A(t_r, t_l)$  of like and unlike strangeness events measured by the CPLEAR Collaboration [12] and discussed in their paper. The proper times at which the strangeness of the neutral kaons is measured at the right and the left of the  $K^0\bar{K}^0$  source at rest are denoted by  $t_r$  and  $t_l$ , respectively. We investigate the dependence of  $A(t_r, t_l)$  on the basis chosen in the 2-dimensional  $K^0\bar{K}^0$  space and on the decoherence parameter  $\zeta$ . Then we use the results on  $A(t_r, t_l)$  of the CPLEAR experiment [12] to estimate the value of the decoherence parameter for two basis choices: first we consider the basis given by  $\{K_L, K_S\}$ , which was used in Ref. [12] to compare spontaneous factorization of the wave function with the result of QM, and, secondly, the basis  $\{K^0, \bar{K}^0\}$ . As we will see, numerically the two estimates of  $\zeta$  will be quite different and, therefore, the statement that spontaneous factorization is excluded with a CL of more than 99.99% is valid only for the  $\{K_L, K_S\}$  basis in the factorization process. Also we will see that this basis is the only one which leads to a vanishing asymmetry upon spontaneous factorization.

Let us mention that strangeness in the  $K^0\bar{K}^0$  system is a quantity analogous to polarization in the well-known entangled two-photon systems. However, strangeness is time-dependent due to  $K^0 \leftrightarrow \bar{K}^0$  oscillations and it has been shown that because of the actual values of the decay constants of  $K_L$  and  $K_S$  it is not possible to find an experimental set-up for violating an inequality of the Bell type [15–17] by using the strangeness of the kaons. This is different from situations where Bell inequalities for kaon systems can be tested with kaon decay products [18]. Finally, we want to stress that the  $K^0\bar{K}^0$  system considered here is a further example exhibiting QM interference effects over macroscopic distances with the neutral kaons being separated several centimeters when their strangeness is measured in the CPLEAR experiment.

## II. A FORMALISM FOR THE $K^0\bar{K}^0$ SYSTEM IN AN ARBITRARY BASIS AND WITH MODIFIED INTERFERENCE TERMS

A  $K^0\bar{K}^0$  pair, created in a  $J^{PC} = 1^{--}$  quantum state and thus antisymmetric under C and P, is described at proper times  $t_r = t_l = 0$  by the an entangled state:

$$|\psi(0,0)\rangle = \frac{1}{\sqrt{2}} \{ |K^0\rangle_r \otimes |\bar{K}^0\rangle_l - |\bar{K}^0\rangle_r \otimes |K^0\rangle_l \} . \quad (2.1)$$

Here  $r$  and  $l$  denote the neutral kaons on the right and left side of the source. On the other hand, the physical states which decay are the long and short-lived states:

$$\begin{aligned} |K_S\rangle &= p|K^0\rangle - q|\bar{K}^0\rangle, \\ |K_L\rangle &= p|K^0\rangle + q|\bar{K}^0\rangle, \end{aligned} \quad (2.2)$$

where the normalization  $|p|^2 + |q|^2 = 1$  is understood. Rewriting the initial state (2.1) in the basis  $\{K_L, K_S\}$ , we obtain

$$|\psi(0, 0)\rangle = \frac{1}{2\sqrt{2}pq} \{ |K_S\rangle_r \otimes |K_L\rangle_l - |K_L\rangle_r \otimes |K_S\rangle_l \}. \quad (2.3)$$

Of course, with respect to QM the states (2.1) and (2.3) are identical and will lead to equal probabilities. But if we modify interference terms by introducing the decoherence parameter  $\zeta$  the derived probabilities depend on the basis chosen [13], a feature discussed already for the analogously entangled  $B^0\bar{B}^0$  state in Refs. [9–11].

To develop a general formalism for the neutral kaons we take an arbitrary basis [11]

$$|k_j\rangle = S_{1j}|K^0\rangle + S_{2j}|\bar{K}^0\rangle \quad \text{with} \quad j = 1, 2, \quad (2.4)$$

where  $S_{ij}$  are elements of an arbitrary invertible matrix  $S$ . Thus the two special cases considered above correspond to  $S = \mathbf{1}$  and  $S = M$  with

$$M = \begin{pmatrix} p & p \\ q & -q \end{pmatrix}, \quad (2.5)$$

respectively.

According to the Wigner–Weisskopf approximation the decaying states evolve exponentially in time:

$$\begin{aligned} |K_S(t)\rangle &= g_S(t)|K_S\rangle, \\ |K_L(t)\rangle &= g_L(t)|K_L\rangle \end{aligned} \quad (2.6)$$

with

$$g_{S,L}(t) = e^{-i\lambda_{S,L}t} \quad \text{and} \quad \lambda_{S,L} = m_{S,L} - \frac{i}{2}\Gamma_{S,L}. \quad (2.7)$$

The subsequent time evolution for  $K^0$  and  $\bar{K}^0$  is therefore given by

$$\begin{aligned} |K^0(t)\rangle &= g_+(t)|K^0\rangle + \frac{q}{p}g_-(t)|\bar{K}^0\rangle, \\ |\bar{K}^0(t)\rangle &= \frac{p}{q}g_-(t)|K^0\rangle + g_+(t)|\bar{K}^0\rangle \end{aligned} \quad (2.8)$$

with

$$g_{\pm}(t) = \frac{1}{2} [\pm e^{-i\lambda_S t} + e^{-i\lambda_L t}]. \quad (2.9)$$

Defining the time evolution matrix by

$$T(t) \equiv M\hat{g}(t)M^{-1}S \quad \text{with} \quad \hat{g} = \text{diag}(g_L, g_S) \quad (2.10)$$

and

$$M\hat{g}(t)M^{-1} = \begin{pmatrix} g_+(t) & \frac{p}{q}g_-(t) \\ \frac{q}{p}g_-(t) & g_+(t) \end{pmatrix} \quad (2.11)$$

we can write the time evolution of the basis vectors (2.4) as

$$|k_j(t)\rangle = T_{1j}(t)|K^0\rangle + T_{2j}(t)|\bar{K}^0\rangle \quad \text{for} \quad j = 1, 2. \quad (2.12)$$

Then the initial state given by (2.1) or (2.3), written in terms of an arbitrary basis (2.4) as

$$|\psi(0, 0)\rangle = \frac{1}{\sqrt{2} \det S} \{ |k_1\rangle_r \otimes |k_2\rangle_l - |k_2\rangle_r \otimes |k_1\rangle_l \}, \quad (2.13)$$

has the time evolution

$$|\psi(t_r, t_l)\rangle = \frac{1}{\sqrt{2} \det S} \{ T_{i1}(t_r)T_{j2}(t_l) - T_{i2}(t_r)T_{j1}(t_l) \} |\hat{k}_i\rangle \otimes |\hat{k}_j\rangle, \quad (2.14)$$

where we have defined

$$|\hat{k}_1\rangle \equiv |K^0\rangle, \quad |\hat{k}_2\rangle \equiv |\bar{K}^0\rangle. \quad (2.15)$$

In Eq.(2.14) a summation over equal indices is understood. At this point we want to stress that QM is invariant under the basis manipulations we have made up to now, as has been explicitly demonstrated in the case of the  $B^0\bar{B}^0$  system in Ref. [11].

Now we are ready to modify QM. The class of observables we are interested in is the probability that the state  $\psi$  evolves into final states  $f_1$  and  $f_2$ , which are measured at proper times  $t_r$  on the right side and  $t_l$  on the left side of the  $K^0\bar{K}^0$  source. This probability is given by

$$\begin{aligned} |\langle f_1 \otimes f_2 | \psi(t_r, t_l) \rangle|^2 &= \frac{1}{2|\det S|^2} \\ &\times \{ |\langle f_1 | k_1(t_r) \rangle|^2 |\langle f_2 | k_2(t_l) \rangle|^2 + |\langle f_1 | k_2(t_r) \rangle|^2 |\langle f_2 | k_1(t_l) \rangle|^2 \\ &- 2(1 - \zeta) \text{Re}(\langle f_1 | k_1(t_r) \rangle^* \langle f_2 | k_2(t_l) \rangle^* \langle f_1 | k_2(t_r) \rangle \langle f_2 | k_1(t_l) \rangle) \}, \end{aligned} \quad (2.16)$$

where the usual QM interference term has been modified by the factor  $1 - \zeta$ . QM corresponds to  $\zeta = 0$  and for  $\zeta = 1$  no interference term is present, corresponding to Furry's hypothesis of spontaneous factorization [13] (called "method A" in his paper). The aim of our investigation is to find the range of  $\zeta$  allowed by present experimental data. We will take the results of experiment of the CPLEAR Collaboration [12] where they measured four final states of the neutral K-meson pairs  $(f_1, f_2) = (K^0, K^0), (\bar{K}^0, \bar{K}^0), (K^0, \bar{K}^0), (\bar{K}^0, K^0)$ . With these states we obtain the following probabilities.

**Like-strangeness events:** final states  $(K^0, K^0)$  and  $(\bar{K}^0, \bar{K}^0)$

$$\begin{aligned}
P_{\zeta}^S(K^0, t_r; K^0, t_l) &= \frac{1}{8} \left| \frac{p}{q} \right|^2 P_{\text{like}}^{\text{QM}}(t_r, t_l) + \zeta \frac{1}{|\det S|^2} \text{Re} \{ (T_{11}^*(t_r) T_{12}(t_r)) (T_{11}^*(t_l) T_{12}(t_l))^* \} , \\
P_{\zeta}^S(\bar{K}^0, t_r; \bar{K}^0, t_l) &= \frac{1}{8} \left| \frac{q}{p} \right|^2 P_{\text{like}}^{\text{QM}}(t_r, t_l) + \zeta \frac{1}{|\det S|^2} \text{Re} \{ (T_{22}^*(t_r) T_{21}(t_r))^* (T_{22}^*(t_l) T_{21}(t_l)) \} .
\end{aligned} \tag{2.17}$$

**Unlike-strangeness events:** final states  $(K^0, \bar{K}^0)$  and  $(\bar{K}^0, K^0)$

$$\begin{aligned}
P_{\zeta}^S(K^0, t_r; \bar{K}^0, t_l) &= \frac{1}{8} P_{\text{unlike}}^{\text{QM}}(t_r, t_l) + \zeta \frac{1}{|\det S|^2} \text{Re} \{ T_{11}^*(t_r) T_{12}(t_r) T_{22}^*(t_l) T_{21}(t_l) \} , \\
P_{\zeta}^S(\bar{K}^0, t_r; K^0, t_l) &= \frac{1}{8} P_{\text{unlike}}^{\text{QM}}(t_r, t_l) + \zeta \frac{1}{|\det S|^2} \text{Re} \{ T_{22}^*(t_r) T_{21}(t_r) T_{11}^*(t_l) T_{12}(t_l) \} .
\end{aligned} \tag{2.18}$$

The QM probabilities in Eqs.(2.17) and (2.18), apart from CP-violating effects, are given by [6]

$$\begin{aligned}
P_{\text{like}}^{\text{QM}} &= 2 e^{-\Gamma(t_r+t_l)} \left\{ \cosh\left(\frac{1}{2}\Delta\Gamma\Delta t\right) - \cos(\Delta m \Delta t) \right\} , \\
P_{\text{unlike}}^{\text{QM}} &= 2 e^{-\Gamma(t_r+t_l)} \left\{ \cosh\left(\frac{1}{2}\Delta\Gamma\Delta t\right) + \cos(\Delta m \Delta t) \right\}
\end{aligned} \tag{2.19}$$

with

$$\Delta t \equiv t_r - t_l, \quad \Delta m \equiv m_L - m_S, \quad \Delta\Gamma = \Gamma_L - \Gamma_S \quad \text{and} \quad \Gamma = \frac{1}{2}(\Gamma_L + \Gamma_S). \tag{2.20}$$

In contrast to the QM terms (2.19), the “decoherence terms” proportional to  $\zeta$  in Eqs.(2.17) and (2.18) depend on the matrix  $S$ , ie., on the choice of the basis. These terms can be characterized by the two functions

$$\begin{aligned}
T_{11}^*(t) T_{12}(t) &= I_+(t) S_{11}^* S_{12} + I_-(t) \left| \frac{p}{q} \right|^2 S_{21}^* S_{22} + I_{+-}(t) \frac{p}{q} S_{11}^* S_{22} + I_{-+}(t) \frac{p^*}{q^*} S_{21}^* S_{12} , \\
T_{22}^*(t) T_{21}(t) &= I_+(t) S_{22}^* S_{21} + I_-(t) \left| \frac{q}{p} \right|^2 S_{12}^* S_{11} + I_{+-}(t) \frac{q}{p} S_{22}^* S_{11} + I_{-+}(t) \frac{q^*}{p^*} S_{12}^* S_{21} ,
\end{aligned} \tag{2.21}$$

where the t-dependent functions are defined by

$$\begin{aligned}
I_{\pm}(t) &= |g_{\pm}(t)|^2 = \frac{1}{2} e^{-\Gamma t} \left\{ \cosh\left(\frac{1}{2}\Delta\Gamma t\right) \pm \cos(\Delta m t) \right\} , \\
I_{+-}(t) &= g_+^*(t) g_-(t) = -\frac{1}{2} e^{-\Gamma t} \left\{ \sinh\left(\frac{1}{2}\Delta\Gamma t\right) + i \sin(\Delta m t) \right\} , \\
I_{-+}(t) &= (I_{+-}(t))^* .
\end{aligned} \tag{2.22}$$

The quantity which is directly sensitive to the interference terms is the asymmetry

$$A(t_r, t_l) = \frac{P_{\text{unlike}}(t_r, t_l) - P_{\text{like}}(t_r, t_l)}{P_{\text{unlike}}(t_r, t_l) + P_{\text{like}}(t_r, t_l)} \tag{2.23}$$

measured in the CPLEAR experiment [12]. With the definition

$$\rho = \frac{1}{2} \left( \left| \frac{p}{q} \right|^2 + \left| \frac{q}{p} \right|^2 \right), \quad (2.24)$$

the QM result for the asymmetry (2.23) is given by

$$A^{\text{QM}}(t_r, t_l) = \frac{(1 - \rho) \cosh(\frac{1}{2}\Delta\Gamma\Delta t) + (1 + \rho) \cos(\Delta m\Delta t)}{(1 + \rho) \cosh(\frac{1}{2}\Delta\Gamma\Delta t) + (1 - \rho) \cos(\Delta m\Delta t)}, \quad (2.25)$$

whereas with the modified interference term the asymmetry depends on  $S$  and  $\zeta$  and has to be calculated with the full expressions (2.17) and (2.18), i.e.,

$$A_\zeta^S(t_r, t_l) = \frac{P_\zeta^S(K^0, t_r; \bar{K}^0, t_l) + P_\zeta^S(\bar{K}^0, t_r; K^0, t_l) - P_\zeta^S(K^0, t_r; K^0, t_l) - P_\zeta^S(\bar{K}^0, t_r; \bar{K}^0, t_l)}{P_\zeta^S(K^0, t_r; \bar{K}^0, t_l) + P_\zeta^S(\bar{K}^0, t_r; K^0, t_l) + P_\zeta^S(K^0, t_r; K^0, t_l) + P_\zeta^S(\bar{K}^0, t_r; \bar{K}^0, t_l)}. \quad (2.26)$$

### III. DISCUSSION OF THE ASYMMETRY

CP violation in  $K^0\bar{K}^0$  mixing is small and proportional to  $\text{Re } \varepsilon \approx (|p/q| - 1)/2$ . In the strangeness asymmetry (2.25) calculated according to the rules of QM, CP violation enters through the coefficient  $\rho$  (2.24) and appears therefore quadratically in the CP-violating parameter. One can easily check that the same suppression of CP-violating effects is true for the asymmetry (2.26) in the bases  $\{K^0, \bar{K}^0\}$  and  $\{K_L, K_S\}$  corresponding to  $S = \mathbf{1}$  and  $S = M$ , respectively. Because of the smallness of CP violation in  $K^0\bar{K}^0$  mixing,  $\rho = 1$  or  $(|p|^2 - |q|^2)^2 = 0$  holds to an extremely good approximation. Though in general the CP-violating parameter appears linearly in Eq.(2.26) we will neglect CP violation in  $K^0\bar{K}^0$  mixing from now on, which is sufficient for our purpose of estimating the allowed range of  $\zeta$  for different bases. Without loss of generality we will use the phase convention  $p = q$ .

As a further simplification we will restrict ourselves to unitary matrices  $S$ . Since the expressions (2.17) and (2.18) do not depend a phase factor multiplying  $S$  we simply assume that

$$S = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix} \in \text{SU}(2) \quad \text{with} \quad |a|^2 + |b|^2 = 1. \quad (3.1)$$

Note that with this convention the basis  $\{K_L, K_S\}$  is described by a matrix  $S$  given by  $a = b = i/\sqrt{2}$ .

In conventional  $K^0\bar{K}^0$  physics, a non-zero difference in the numbers of  $K^0$  and  $\bar{K}^0$  pairs measured at  $t = t_r = t_l$  signifies CP violation. Since we assume CP conservation in  $K^0\bar{K}^0$  mixing we want to ask the question if the artificial modification (2.16) of the QM interference terms introduces such a difference. Indeed, in general this is the case as seen from the equation

$$\begin{aligned} & P_\zeta^S(K^0, t; K^0, t) - P_\zeta^S(\bar{K}^0, t; \bar{K}^0, t) \\ &= -2\zeta e^{-2\Gamma t} \sinh\left(\frac{1}{2}\Delta\Gamma t\right) \{(|a|^2 - |b|^2) \text{Re}(ab) \cos(\Delta m t) + \text{Im}(a^2 b^2) \sin(\Delta m t)\}. \end{aligned} \quad (3.2)$$

However, for the bases  $\{K_L, K_S\}$  with  $|a| = |b|$  and  $a^2, b^2$  being real and  $\{K^0, \bar{K}^0\}$  with  $b = 0$  the right-hand side of Eq.(3.2) is zero (see Ref. [11] for an analogous consideration in the  $B^0 \bar{B}^0$  system).

With the above simplifying assumptions we can readily calculate the asymmetry (2.26) for any orthonormal basis given by a unitary matrix  $S$  and for any decoherence parameter  $\zeta$ . For  $\zeta = 0$  we obtain the QM result [12]

$$A^{\text{QM}}(t_r, t_l) = \frac{\cos(\Delta m \Delta t)}{\cosh(\frac{1}{2}\Delta\Gamma\Delta t)}, \quad (3.3)$$

which is, of course, independent of  $S$ . For the basis  $\{K_L, K_S\}$  we arrive at the simple formula

$$A_\zeta^{K_L K_S}(t_r, t_l) = (1 - \zeta) A^{\text{QM}}(t_r, t_l), \quad (3.4)$$

whereas for  $\{K^0, \bar{K}^0\}$  we obtain

$$A_\zeta^{K^0 \bar{K}^0}(t_r, t_l) = \frac{\cos(\Delta m \Delta t) - \frac{1}{2}\zeta\{\cos(\Delta m \Delta t) - \cos(\Delta m(t_r + t_l))\}}{\cosh(\frac{1}{2}\Delta\Gamma\Delta t) - \frac{1}{2}\zeta\{\cosh(\frac{1}{2}\Delta\Gamma\Delta t) - \cosh(\frac{1}{2}\Delta\Gamma(t_r + t_l))\}}. \quad (3.5)$$

Eqs.(3.4) and (3.5) represent the two cases for which we will perform a numerical estimate of the decoherence parameter  $\zeta$  in the next section.

In our formalism, Furry's hypothesis ("method A" in Ref. [13]) corresponds to  $\zeta = 1$ . Looking at Eqs.(3.4) and (3.5) we read off in this case

$$A_1^{K_L K_S}(t_r, t_l) = 0, \quad (3.6)$$

$$A_1^{K^0 \bar{K}^0}(t_r, t_l) = \frac{\cos(\Delta m \Delta t) + \cos(\Delta m(t_r + t_l))}{\cosh(\frac{1}{2}\Delta\Gamma\Delta t) + \cosh(\frac{1}{2}\Delta\Gamma(t_r + t_l))}. \quad (3.7)$$

In the basis  $\{K_L, K_S\}$  the asymmetry vanishes [12], whereas in the basis  $\{K^0, \bar{K}^0\}$  spontaneous factorization leads to a non-zero asymmetry (3.7). One might ask the question for which bases the asymmetry (2.26) is zero at  $\zeta = 1$  for arbitrary  $t_r, t_l$ . The answer to this question is given by the following proposition.

**Proposition:** *With the assumptions  $S \in SU(2)$  and  $|p/q| = 1$ , Furry's hypothesis leads to a zero asymmetry (2.26) if and only if spontaneous factorization of the  $K^0 \bar{K}^0$  wave function takes place in the  $\{K_L, K_S\}$  basis.*

*Proof:* We study the general expression

$$\begin{aligned} & P_\zeta^S(K^0, t_r; \bar{K}^0, t_l) + P_\zeta^S(\bar{K}^0, t_r; K^0, t_l) - P_\zeta^S(K^0, t_r; K^0, t_l) - P_\zeta^S(\bar{K}^0, t_r; \bar{K}^0, t_l) \\ &= e^{-\Gamma(t_r+t_l)} \left\{ \cos(\Delta m \Delta t) \right. \\ &\quad - \frac{1}{2}\zeta \left[ (|a^*|^2 + b^2)^2 + 4|a|^2|b|^2 \cos(\Delta m \Delta t) \right. \\ &\quad \left. - (|a^*|^2 + b^2)^2 - 4|a|^2|b|^2 \cos(\Delta m(t_r + t_l)) \right. \\ &\quad \left. \left. - 4(|a|^2 - |b|^2) \text{Im}(ab) \sin(\Delta m(t_r + t_l)) \right] \right\}. \end{aligned} \quad (3.8)$$

The right-hand side of the equation is zero  $\forall t_r, t_l$  at  $\zeta = 1$  if the system of equations

$$\begin{aligned} |(a^*)^2 + b^2|^2 + 4|a|^2|b|^2 &= 2, \\ |(a^*)^2 + b^2|^2 - 4|a|^2|b|^2 &= 0, \\ (|a|^2 - |b|^2) \operatorname{Im}(ab) &= 0 \end{aligned} \quad (3.9)$$

is fulfilled. The general solution of this system is given by  $a = e^{i\alpha}/\sqrt{2} = \epsilon b^*$  with  $\epsilon = \pm 1$  and  $\alpha$  being an arbitrary phase. It is then easy to show that a matrix  $S$  with these coefficients represents the basis  $\{K_L, K_S\}$  apart from trivial redefinitions.  $\square$

#### IV. ESTIMATION OF THE DECOHERENCE PARAMETER $\zeta$ FROM THE DATA OF THE CPLEAR EXPERIMENT

In the CPLEAR experiment [12] at CERN,  $K^0 \bar{K}^0$  pairs are produced by  $p\bar{p}$  annihilation at rest. These pairs are predominantly in an antisymmetric state with quantum numbers  $J^{PC} = 1^{--}$  and the strangeness of the kaons is detected via strong interactions in surrounding absorbers. The experimental set-up has two configurations. In configuration C(0) both kaons have nearly equal proper times ( $t_r \approx t_l$ ) when they interact in the absorber. This fulfills the conditions for an EPR-type experiment. In configuration C(5) the flight-path difference is 5 cm on average, corresponding to a proper time difference  $|t_r - t_l| \approx 1.2 \tau_S$ .

The asymmetry (2.23) is measured for these two configurations, giving the experimental results [12]  $A^{\text{exp}}(0) = 0.81 \pm 0.17$  for C(0) and  $A^{\text{exp}}(5) = 0.48 \pm 0.12$  for C(5). On the other hand, the QM predictions for the asymmetry, when corrected according to the specific experimental design, yield  $A_{\text{corr}}^{\text{QM}}(0) = 0.93$  and  $A_{\text{corr}}^{\text{QM}}(5) = 0.56$ . These values are in agreement with the above experimental ones, demonstrating in this way the non-separability of the QM  $K^0 \bar{K}^0$  wavefunction.

Our approach is somewhat different. We estimate the amount of decoherence and show, as already emphasized, its dependence on the basis chosen. Specifically, we fit the decoherence parameter  $\zeta$  by comparing the asymmetry for the two cases (3.4) and (3.5) with the experimental data of CPLEAR. For the fit of the parameter  $\zeta$  we use the least squares method of Ref. [19] which is an effective variance method, taking into account not only the experimental error in  $A(t_r, t_l)$  but also the variations in  $x_{r,l} = 4.28 \text{ cm } (t_{r,l}/\tau_S)$ , the space coordinates where the strangeness of the kaons is measured.

First we discuss the asymmetry in the  $\{K_L, K_S\}$  basis where formula (3.4) is relevant. This case is rather easy to handle because  $\zeta$  enters into the asymmetry linearly. Our fit result is

$$\bar{\zeta} = 0.13^{+0.16}_{-0.15}. \quad (4.1)$$

The CL for this fit is 97%.

In Fig. 1 we have plotted the asymmetry (3.4) for the values of  $\zeta$  given in Eq.(4.1). The three solid lines represent the asymmetry (3.4) for the meanvalue  $\bar{\zeta}$  and its one standard deviation values  $\bar{\zeta} = 0.29$  and  $-0.03$  (see Eq.(4.1)), respectively. The dashed curve shows the QM prediction (3.3) ( $\zeta = 0$ ). The experimental results for the two configurations C(0)



and C(5) are also indicated according to Fig. 9 in Ref. [12], where in comparison to the experimental numbers quoted above some “background subtraction” has been performed [12]. We have scaled the variable  $\Delta t$  in the QM asymmetry (3.3) in order to reproduce the QM curve in Fig. 9 of the CPLEAR Collaboration [12].

The case of the asymmetry in the  $\{K^0, \bar{K}^0\}$  basis (3.5) is a bit more intricate than the previous one because formula (3.5) depends not only on the time difference  $\Delta t = t_r - t_l$  but also on the sum  $t_r + t_l$ , and, moreover, the dependence on  $\zeta$  is not linear. We estimate the values of the sum  $t_r + t_l$  and their variations from the experimental configurations C(0) and C(5). With this additional but less accurate information we obtain the estimate

$$\bar{\zeta} \sim 0.4 \pm 0.7. \quad (4.2)$$

Here the quality of the fit is also good with a CL of about 67%.

In order to assess the validity of QM there are two measures associated with the parameter  $\zeta$ . They are given by the distance of the fitted mean value from 0 and from 1, each expressed in units of one standard deviation, corresponding to pure QM and to Furry’s hypothesis, respectively. Considering first the distance of  $\bar{\zeta}$  from 0, we see that for both bases,  $\{K_L, K_S\}$  and  $\{K^0, \bar{K}^0\}$ , the results of our fit are very well compatible with QM. On the other hand,  $\zeta = 1$  corresponds to Furry’s hypothesis, i.e., total decoherence or spontaneous factorization. In this case, with the  $\{K_L, K_S\}$  basis the asymmetry vanishes identically (see Eq.(3.6)), which – as found by the CPLEAR Collaboration [12] – has a probability of less than  $10^{-4}$ . In the  $\{K^0, \bar{K}^0\}$  basis, however, formula (3.7) gives the values

$$A_1^{K^0\bar{K}^0}(0) = 0.90, \quad A_1^{K^0\bar{K}^0}(5) = 0.50 \quad (4.3)$$

for the two configurations C(0) and C(5), respectively, which are within one standard deviation of the corresponding data and therefore not excluded at all. Note also that the two fit results for  $\zeta$  (4.1) and (4.2) clearly disfavour negative values of the decoherence parameter.

Last but not least we want to mention that we also checked the effect of the experimental errors of the input quantities  $\Delta m$  and  $\Delta \Gamma$  on the asymmetries (3.4) and (3.5). The resulting error in the asymmetries is less than 1.5% for all times. Therefore, we can safely neglect these errors for our purpose.

## V. CONCLUSIONS

We reconsidered the results of the experiment of the CPLEAR Collaboration [12] which investigated strangeness correlations of  $K^0\bar{K}^0$  pairs produced by  $p\bar{p}$  annihilation. We introduced the decoherence parameter  $\zeta$  in order to measure quantitatively deviations from pure QM. This introduces a certain arbitrariness since the quantity we consider, the asymmetry (2.26), depends strongly on the basis chosen (pure QM, of course, not). This is not so surprising, it is merely the consequence of the freedom in QM to choose a basis in  $K^0-\bar{K}^0$  space, which eventually leads to different interference terms for different basis choices [10,11]. Thus the parameters  $\zeta$  modifying these different interference terms will get in general different values when confronted with experiment. What is essential, however, is the existence of a basis where the  $K^0\bar{K}^0$  system is far away from total decoherence and the corresponding  $\zeta$

is close to 0 in agreement with QM. We have shown that the “best basis” in this respect is the  $\{K_L, K_S\}$  basis (see proposition).

Furthermore, because of the introduction of the matrix  $S$ , which is arbitrary apart from  $S \in \text{SU}(2)$ , even for  $|p/q| = 1$  we have in general  $P_\zeta^S(K^0, t; K^0, t) - P_\zeta^S(\bar{K}^0, t; \bar{K}^0, t) \neq 0$ , a sign for CP violation in  $K^0 \bar{K}^0$  mixing. Thus, in our ad hoc modification of QM by the decoherence parameter  $\zeta$ , the matrix  $S$  mimics CP violation in general. However, as we have shown, for the two bases under consideration,  $\{K_L, K_S\}$  and  $\{K^0, \bar{K}^0\}$ , the difference (3.2) vanishes. This is analogous to considerations in the  $B^0 \bar{B}^0$  system [11].

Finally, we want to mention that our statistical analysis is not very refined and, in addition, further experimental data could diminish the errors of our fits. Nevertheless, long range interference effects, i.e., the presence of the QM interference term in the  $K^0 \bar{K}^0$  system, are quite well confirmed in agreement with QM and thus also interference effects of massive particles over macroscopic distances.

## REFERENCES

- [1] A. Einstein, B. Podolski and N. Rosen, Phys. Rev. **47**, 777 (1935).
- [2] J.S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1987).
- [3] A. Aspect, J. Dalibard and G. Roger, Phys. Rev. Lett. **49**, 1840 (1982).
- [4] P.G. Kwiat, K. Mattle, H. Weinfurter and A. Zeilinger, Phys. Rev. Lett. **75**, 4337 (1995); G. Weihs, T. Jennewein, C. Simon, H. Weinfurter and A. Zeilinger, Phys. Rev. Lett. **81**, 5039 (1998).
- [5] T.D. Lee and N.C. Yang, reported by T.D. Lee at Argonne National Laboratory, May 28, 1960 (unpublished); D.R. Inglis, Rev. Mod. Phys. **33**, 1 (1961); T.B. Day, Phys. Rev. **121**, 1204 (1961).
- [6] J. Six, Phys. Lett. B **114**, 200 (1982).
- [7] F. Selleri, Lett. Nuovo Cim. **36**, 521 (1983); P. Privitera and F. Selleri, Phys. Lett. B **296**, 261 (1992).
- [8] A. Datta and D. Home, Phys. Lett. A **119**, 3 (1986).
- [9] R.A. Bertlmann and W. Grimus, Phys. Lett. B **392**, 426 (1997).
- [10] G.V. Dass and K.V.L. Sarma, Eur. Phys. J. C **5**, 283 (1998).
- [11] R.A. Bertlmann and W. Grimus, Phys. Rev. D **58**, 034014 (1998).
- [12] A. Apostolakis *et al.*, CPLEAR Coll., Phys. Lett. B **422**, 339 (1998).
- [13] W.H. Furry, Phys. Rev. **49**, 393 (1936).
- [14] P.H. Eberhard, in *The Second DAΦne Physics Handbook*, edited by L. Maiani, G. Pancheri and N. Paver (SIS-Pubblicazioni dei Laboratori di Frascati, Italy, 1995) Vol I, p. 99.
- [15] G.C. Ghirardi, R. Grassi und T. Weber, in Proc. of Workshop on *Physics and Detectors for DAΦNE, the Frascati Φ Factory*, April 9-12th, 1991, edited by G. Pancheri.
- [16] G.C. Ghirardi, R. Grassi and R. Ragazzon, in *The DAΦNE Physics Handbook*, edited by L. Maiani, G. Pancheri and N. Paver (Servizio Documentazione dei Laboratori Nazionale di Frascati, Italy, 1992), Vol. I, p. 283.
- [17] B. Hiesmayr, Diploma Thesis (1999).
- [18] P.H. Eberhard, Nucl. Phys. B **398**, 155 (1993); A. Di Domenico, Nucl. Phys. B **450**, 293 (1995); F. Benatti and R. Floreanini, Phys. Rev. D **57**, 1332 (1998) and hep-ph/9812353; B. Ancochea, A. Bramon and M. Nowakowski, preprint UAB-FT-459-98 (hep-ph/9811404); A. Bramon and M. Nowakowski, UAB-FT-458-98 (hep-ph/9811406).
- [19] J. Orear, Am. J. Phys. **50**, 912 (1982).

# FIGURES

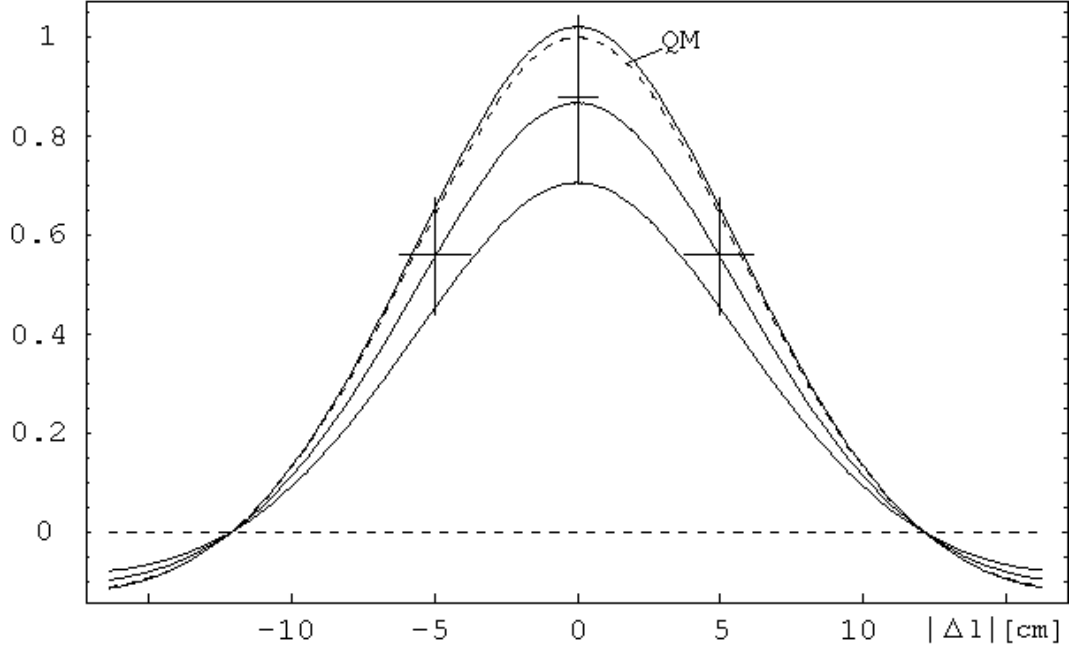


FIG. 1. The asymmetry (3.4) as a function of the difference in the distances travelled by the kaons to the points where their strangeness is measured. The dashed curve corresponds to QM with the decoherence parameter  $\zeta = 0$ , whereas the solid curves correspond to the values of  $\zeta$  obtained by the fit (4.1) to the CPLEAR data. The two data points represented by the crosses have been taken from Fig. 9 of Ref. [12]. The horizontal dashed line indicates the zero asymmetry for  $\zeta = 1$  (3.6), the consequence of Furry's hypothesis with respect to the  $\{K_L, K_S\}$  basis.